Windows as Renewable Energy Sources for Europe
Window Energy Data Network

Project supported by DG for Energy and Transport of the European Commission

www.windat.org

contract NNE5-2000-122

A Method for Characterizing the Thermal Properties of Windows Frame Profiles.

Frank Pedersen
Jacob Birck Laustsen
Svend Svendsen

Technical University of Denmark
Department of Civil Engineering
Brovej, Building 118
DK-2800 Kgs. Lyngby

WinDat document N2.09 - Public **September 2003**

This document was produced within WinDat, with active contribution by the members of this European Thematic Network. See www.windat.org for more information.

Contact:

On this report:

The authors:
Email: Frank Pedersen fp@byg.dtu.dk
Jacob Birck Laustsen jbl@byg.dtu.dk
Svend Svendsen ss@byg.dtu.dk

On Thematic Network WinDat:

WinDat coordinator: Dick van Dijk, TNO Building and Construction Research, Delft, The Netherlands
Email: H.vanDijk@bouw.tno.nl

Research funded in part by
THE EUROPEAN COMMISSION
in the framework of the Community activities
in the Field of the specific programme for RTD and
demonstration on Energy, Environment and Sustainable
Development

BYG•DTU

Frank Pedersen
Jacob Birck Laustsen
Svend Svendsen

WinDat WP2.3
A method for characterizing the thermal properties of windows frame profiles.

TECHNICAL
UNIVERSITY
OF DENMARK

DTU

BYG•DTU
september 2003

Index

1 Introduction

The thermal properties of window frame profiles are traditionally characterized by the linear thermal transmittance (Ψ-value), which is the additional heat flow caused by the interaction of the frame and the glass edge, including the effect of the spacer.

However, changing the overall thermal conductance of the edge construction (L-value), or glass thickness (d), or the thermal transmittance for the center of the glazing unit (U_g-value), changes the overall heat flow pattern through the glazing unit, and thereby changes the linear thermal transmittance of the frame profile.

For windows manufacturers, this means that characterizing the thermal properties of a frame profile, requires calculating one Ψ-value for every combination of L, d and U_g that is used with that profile.

This report describes a method for characterizing the Ψ-value as a function of L, d and U_g. The proposed method does so by fitting a regression model of Ψ to a limited number of detailed calculations of Ψ, made with a 2D-program like Therm or Bisco. If a good match can be established between the model and the detailed calculations of Ψ, the regression model parameters can then be used for characterizing the Ψ-value of the frame profile.

Validating the model can be done by comparing it to other detailed calculations of Ψ. For the regression model to be a valid representation of the detailed calculations of Ψ, the residuals between the model and the validation calculations of Ψ must be insignificant. Since the Ψ-value usually is expressed using 2 digits, this objective is considered to be satisfied, if the absolute values of all residuals are below $0.005\,\mathrm{W/mK}$. The results presented in this report indicate that this objective can be achieved.

2 Modelling the linear thermal transmittance using a thermal network

This section concerns the derivation of a mathematic regression model of the linear thermal transmittance Ψ for window profiles.

2.1 Background

The model will be derived using a thermal network, shown in figure 1.

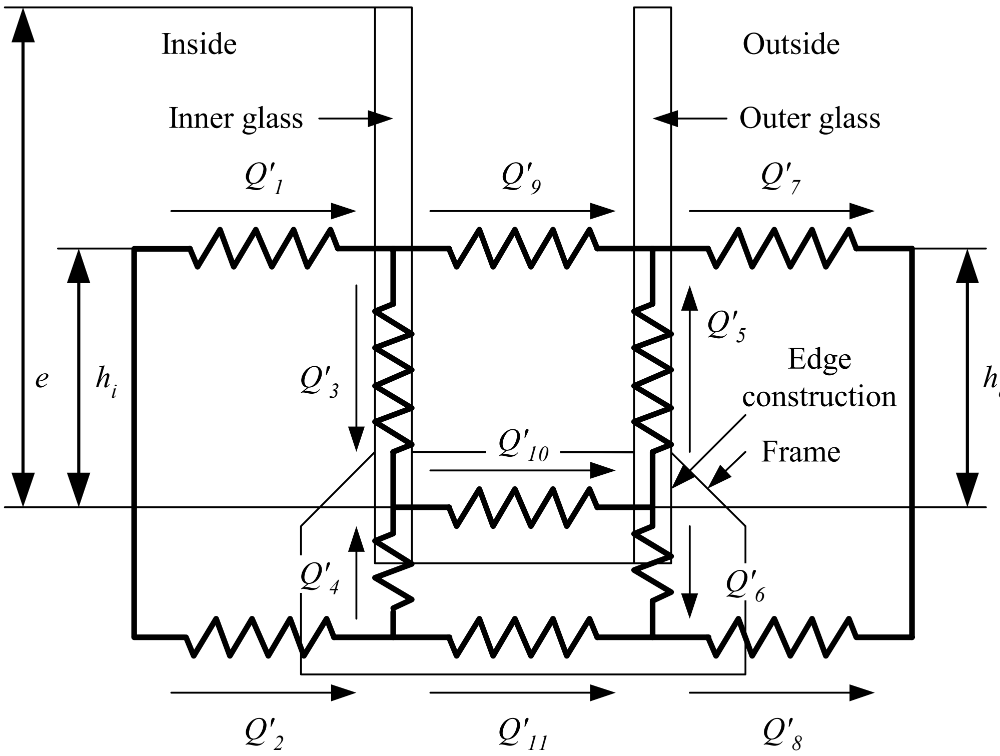

Figure 1. A thermal network model of the heat flows through a window profile. The equivalent heights h_i and h_e, and the edge size parameter e are also shown.

In figure 1, the notation $Q' = \frac{1}{w}\frac{\partial Q}{\partial t}$ has been used, where w is the width of the glazing unit. All heat flows are thus per unit length of the edge (assembly of glazing and frame profile). The equivalent heights h_i and h_e are parameters used for modeling the heat flow parallel to the glass, close to the edge. Finally, the parameter e is the size of the region, where 2-dimensional heat flow is assumed to take place.

Figure 2 shows the resistances and the temperatures in the network. The parameters d and λ are the glass thickness and the heat conductivity of the glass, respectively.

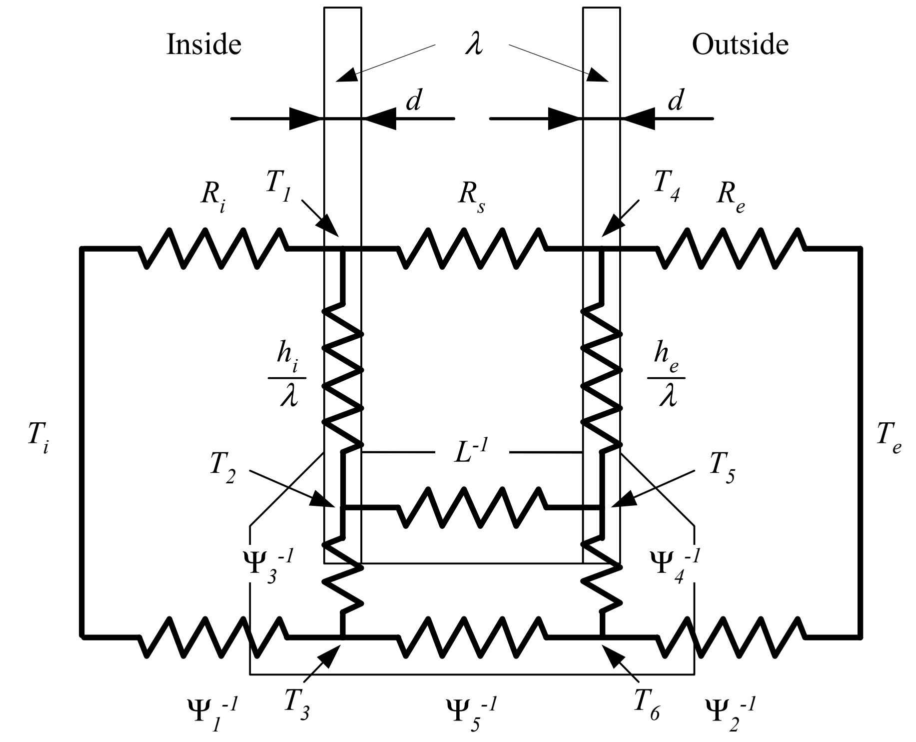

Figure 2. The resistances and the temperatures in the thermal network model.

The network represents a simplification of the actual 2-dimensional heat flow through the window. The total heat flow in the network is:

$$Q'_{\text{tot}} = Q'_7 + Q'_8 \tag{2.1}$$

The linear thermal transmittance for the edge construction represents the additional heat flow through the window caused by the interaction between the frame and the glass edge. It is given as the difference between the total heat flow and the heat flow Q'_{com} through the individual components, when they are not interacting with each other, normalized with the temperature difference. In this case

$$Q'_{\text{com}} = \Psi_f \left(T_i - T_e\right) + U_g e\left(T_i - T_e\right) \tag{2.2}$$

where Ψ_f is the thermal transmittance for the frame profile, and is given by

$$\Psi_f = U_f w_f \tag{2.3}$$

Here, U_f and w_f are the U-value and width of the frame, respectively. The linear thermal transmittance for the edge construction can then be approximated by the following expression:

$$\Psi\left(L,d,U_g\right) \simeq \hat{\Psi}\left(L,d,U_g\right) = \frac{Q'_{\text{tot}} - Q'_{\text{com}}}{T_i - T_e} = \frac{Q'_{\text{tot}}}{T_i - T_e} - \Psi_f - U_g e \tag{2.4}$$

2.2 Deriving the model

In order to derive the model $\hat{\Psi}$, it is necessary to find an expression for Q'_{tot}, in terms of the resistances in the network, which is the concern of this section. A general approach for solving network problems will be used, which applies to all networks consisting of linear components.

The heat capacity of the network components are omitted, therefore no heat is built up in the nodes. This gives the following continuity equations:

$$Q'_1 = Q'_3 + Q'_9 \tag{2.5}$$

$$Q'_3 + Q'_4 = Q'_{10} \tag{2.6}$$

$$Q'_2 = Q'_4 + Q'_{11} \tag{2.7}$$

$$Q'_9 + Q'_5 = Q'_7 \tag{2.8}$$

$$Q'_{10} = Q'_5 + Q'_6 \tag{2.9}$$

$$Q'_{11} + Q'_6 = Q'_8 \tag{2.10}$$

The center U-value of the window needs to be incorporated into the network equations. This value is given by

$$U_g = \frac{1}{R_i + R_s + R_e} \tag{2.11}$$

which gives

$$R_s = U_g^{-1} - R_i - R_e \tag{2.12}$$

Also the thermal transmittance Ψ_f for the frame is needed, given by

$$\frac{1}{\Psi_f} = \frac{1}{\Psi_1} + \frac{1}{\Psi_5} + \frac{1}{\Psi_2} \tag{2.13}$$

which gives

$$\Psi_5 = \left(\frac{1}{\Psi_f} - \frac{1}{\Psi_1} - \frac{1}{\Psi_2} \right)^{-1} \tag{2.14}$$

The heat flows in the network are governed by the following equations:

$$Q'_1 = \frac{e}{R_i}\left(T_i - T_1\right) \tag{2.15}$$

$$Q'_2 = \Psi_1\left(T_i - T_3\right) \tag{2.16}$$

$$Q'_3 = \frac{d\lambda}{h_i}\left(T_1 - T_2\right) \tag{2.17}$$

$$Q'_4 = \Psi_3\left(T_3 - T_2\right) \tag{2.18}$$

$$Q_5' = \frac{d\lambda}{h_e}\left(T_5 - T_4\right) \tag{2.19}$$

$$Q_6' = \Psi_4\left(T_5 - T_6\right) \tag{2.20}$$

$$Q_7' = \frac{e}{R_e}\left(T_4 - T_e\right) \tag{2.21}$$

$$Q_8' = \Psi_2\left(T_6 - T_e\right) \tag{2.22}$$

$$Q_9' = \frac{e}{R_s}\left(T_1 - T_4\right) = \frac{e}{U_g^{-1} - R_i - R_e}\left(T_1 - T_4\right) \tag{2.23}$$

$$Q_{10}' = L\left(T_2 - T_5\right) \tag{2.24}$$

$$Q_{11}' = \Psi_5\left(T_3 - T_6\right) = \left(\frac{1}{\Psi_f} - \frac{1}{\Psi_1} - \frac{1}{\Psi_2}\right)^{-1}\left(T_3 - T_6\right) \tag{2.25}$$

The continuity equations (2.5) to (2.10), and the flow equations (2.15) to (2.25), can be arranged in the following way:

$$\begin{bmatrix}
d_1 & & & & & & & & & & & 1 & & & & & \\
 & d_2 & & & & & & & & & & & & 1 & & & \\
 & & d_3 & & & & & & & & & -1 & 1 & & & & \\
 & & & d_4 & & & & & & & & & 1 & -1 & & & \\
 & & & & d_5 & & & & & & & & & & 1 & -1 & \\
 & & & & & d_6 & & & & & & & & & & -1 & 1 \\
 & & & & & & d_7 & & & & & & & & -1 & & \\
 & & & & & & & d_8 & & & & & & & & & -1 \\
 & & & & & & & & d_9 & & & -1 & & & 1 & & \\
 & & & & & & & & & d_{10} & & & -1 & & & 1 & \\
 & & & & & & & & & & d_{11} & & & -1 & & & 1 \\
1 & & -1 & & & & & & -1 & & & & & & & & \\
 & & 1 & 1 & & & & & & -1 & & & & & & & \\
 & 1 & & -1 & & & & & & & -1 & & & & & & \\
 & & & & 1 & & -1 & & 1 & & & & & & & & \\
 & & & & -1 & -1 & & & & 1 & & & & & & & \\
 & & & & & 1 & & -1 & & & 1 & & & & & &
\end{bmatrix}
\cdot
\begin{bmatrix} Q_1' \\ Q_2' \\ Q_3' \\ Q_4' \\ Q_5' \\ Q_6' \\ Q_7' \\ Q_8' \\ Q_9' \\ Q_{10}' \\ Q_{11}' \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}
=
\begin{bmatrix} T_i \\ T_i \\ 0 \\ 0 \\ 0 \\ 0 \\ -T_e \\ -T_e \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.26}$$

with

$$d_1 = \frac{R_i}{e} \tag{2.27}$$

$$d_2 = \frac{1}{\Psi_1} \tag{2.28}$$

$$d_3 = \frac{h_i}{d\lambda} \tag{2.29}$$

$$d_4 = \frac{1}{\Psi_3} \tag{2.30}$$

$$d_5 = \frac{h_e}{d\lambda} \tag{2.31}$$

$$d_6 = \frac{1}{\Psi_4} \tag{2.32}$$

$$d_7 = \frac{R_e}{e} \tag{2.33}$$

$$d_8 = \frac{1}{\Psi_2} \tag{2.34}$$

$$d_9 = \frac{U_g^{-1} - R_i - R_e}{e} \tag{2.35}$$

$$d_{10} = \frac{1}{L} \tag{2.36}$$

$$d_{11} = \frac{1}{\Psi_f} - \frac{1}{\Psi_1} - \frac{1}{\Psi_2} \tag{2.37}$$

In (2.26), zeros have been omitted from the matrix. In general, the equations governing the flows and potentials in a network with p nodes and q connections have the following structure:

$$\begin{bmatrix} D & C \\ C^{\mathrm{T}} & 0_{p\times p} \end{bmatrix} \cdot \begin{bmatrix} Q' \\ T \end{bmatrix} = \begin{bmatrix} b \\ 0_{p\times 1} \end{bmatrix} \tag{2.38}$$

where

- $C \in \mathbb{R}^{q\times p}$ is the incidence matrix for the interior nodes in the network.
- $D \in \mathbb{R}^{q\times p}$ is a diagonal matrix containing the elements $d_1,\ldots,d_q$.
- $Q' \in \mathbb{R}^{q}$ contains the network flows $Q_1',\ldots,Q_q'$.
- $T \in \mathbb{R}^{p}$ contains the network potentials $T_1',\ldots,T_p'$.
- $b \in \mathbb{R}^{q}$ contains boundary values for flows connected to boundary nodes, and zero otherwise.

In the above, *boundary nodes* refer to nodes where the potentials are known, and *interior nodes* refer to nodes, where the potentials are unknown. The equations (2.38) have the following solution:

$$T = \left(C^{\mathrm{T}} D^{-1} C\right)^{-1} C^{\mathrm{T}} D^{-1} b \tag{2.39}$$

$$Q' = D^{-1}\left(b - CT\right) \tag{2.40}$$

In general, the heat flow Q'_{tot}, defined in (2.1), is a sum of some of the elements in Q', so the model (2.4) for Ψ can be expressed as:

$$\hat{\Psi}\left(L,d,U_g\right)=\frac{v^{\mathrm{T}}Q'}{T_i-T_e}-\Psi_f-U_g e \tag{2.41}$$

where $v\in\mathbb{R}^m$ is a vector of either ones or zeros. In this case,

$$v=\left[0,0,0,0,0,0,1,1,0,0,0\right]^{\mathrm{T}} \tag{2.42}$$

which gives the desired value for Q'_{tot}. In order to calculate the model $\hat{\Psi}$, T is first calculated using (2.39), then Q' is calculated using (2.40), and finally $\hat{\Psi}$ is calculated using (2.41).

Changing L, d, U_g, or any of the resistances defined in figure 2, changes either the elements in the matrix D, or the 1-dimensional contributions in (2.41). For the remainder of this report, the notation

$$\hat{\Psi}\left(x,t\right)=\frac{v^{\mathrm{T}}Q'}{T_i-T_e}-\Psi_f-U_g e \tag{2.43}$$

will be used for the model, where

$$x=\left[h_i,h_e,e,R_i,\Psi_1,\Psi_2,\Psi_3,\Psi_4\right]^{\mathrm{T}} \tag{2.44}$$

is a vector of variable model parameters, and where

$$t=\left[L,d,U_g\right]^{\mathrm{T}} \tag{2.45}$$

is a sample point. The remaining parameters are assumed to be constant, and are in this case assigned the following values:

$$\begin{aligned} T_i &= 1\ \text{K} \\ T_e &= 0\ \text{K} \\ \lambda &= 1\ \text{W/mK} \\ R_e &= 0.04\ \text{m}^2\text{K/W} \\ \Psi_f &= 0.197905\ \text{W/m}^2\text{K} \end{aligned} \tag{2.46}$$

3 Using the network model to represent detailed computations of the linear thermal transmittance

In order to use the model (2.43) to represent detailed calculations of Ψ, the parameters (2.44) must be determined in such a way, that the residuals between the model and the calculations of Ψ are minimal. Section 3.1 concerns the definition of an appropriate data fitting problem. The training and test sets are the topics of section 3.1.

3.1 The data fitting problem

When characterizing profiles using the model parameters (2.44), it is important to ensure, that these parameters are uniquely determined, so that only one set of parameters characterize any given profile. The possibilities for proving uniqueness of the solution to the data fitting problem presented here will be investigated in a future version of this report.

Given a set of m calculations of Ψ for various values of L, d and U_g:

$$S = \{(t_1, \Psi_1), \dots, (t_m, \Psi_m)\} \tag{3.1}$$

the m residuals between the model and the calculations of Ψ are:

$$r_i(x) = \hat{\Psi}(x, t_i) - \Psi_i,\ i = 1, \dots, m \tag{3.2}$$

The network model represents a simplification of the actual heat flows, therefore, some residual can be expected between the model and detailed calculations of Ψ. If the model predicts Ψ-values that are lower than the true Ψ-values, the actual heat loss through the window will be higher than expected, leading to higher heating expenses than expected. On the other hand, if the model predicts Ψ-values that are higher than the true Ψ-values, this might lead to the choice of expensive, high performance window systems, which will increase the expenses for constructing the building.

In this work, a conservative approach is used when fitting the network model to calculations of Ψ: The model is fitted to Ψ in such a way, that it always predicts higher values than the calculations of Ψ.

Therm and Bisco computes Ψ by solving a 2-dimensional heat conduction equation, which can be considered to be a numerically stable process. The detailed calculations of Ψ can therefore be assumed not to contain "outliers", i.e. data points which are severely mismatched with the general tendency of the data.

Therefore, the data fitting problem is not defined as an L_1-problem, which is stable to outliers, or as an L_2-problem, which is a classical way of defining data fitting problems. Instead it is defined as a minimax problem, which makes the solution sensitive to outliers, but in this context it makes sense to find a solution, where the maximum residual between the model and the calculations of Ψ is minimal. These considerations lead to the following definition of the data fitting problem:

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \max_{0 \le i \le m} \{r_i(x)\},\ \text{subject to } r_i(x) \ge 0,\ i = 1, \dots, m \tag{3.3}$$

where n is the number of parameters in the model, in this case, $n = 8$. The solution to (3.3) is denoted x^*. This constrained minimax problem can be solved using the following exact penalty approach:

$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \max_{\substack{0 \le i \le m \\ 0 \le j \le m}} \left\{ r_i(x), r_i(x) - \sigma r_j(x) \right\} \tag{3.4}$$

From the theory of minimax optimization, it is known that there exists a $\hat{\sigma} > 0$, such that for all $\sigma > \hat{\sigma}$, the problems (3.3) and (3.4) have the same solution. The problem (3.3) can therefore be solved using the following approach:

Given $\sigma > 0$
Repeat
  Compute the solution estimate $\hat{x}$ by solving (3.4)
  If any constraint is violated
    $\sigma \leftarrow 10 \cdot \sigma$
  End
Until no constraints are violated

When σ becomes sufficiently large, the solution estimate $\hat{x}$ becomes equal to the true solution x^*. A numerical method for solving unconstrained minimax problems can be found in [3], which is used when solving (3.4).

3.2 The training and test data sets

The linear thermal transmittance Ψ has to be sampled in such a way, that once the model parameters (2.44) are determined, the model (2.43) is valid within some region of interest.

In order to determine and evaluate the model parameters, the following two sets of samples are needed:

The training set. This data set contains m samples of calculations of Ψ, for various values of L, d and U_g, and is used when solving the data fitting problem (3.3).

The test set. This data set contains samples of calculations of Ψ, for other values of L, d and U_g, and is used for validating the model, once the model parameters are determined.

In this work, upper and lower bounds on the parameters define the region of interest. Experience shows, that using the corners of this region gives good estimates of the model parameters. The region of interest and the training set are shown in figure 3.

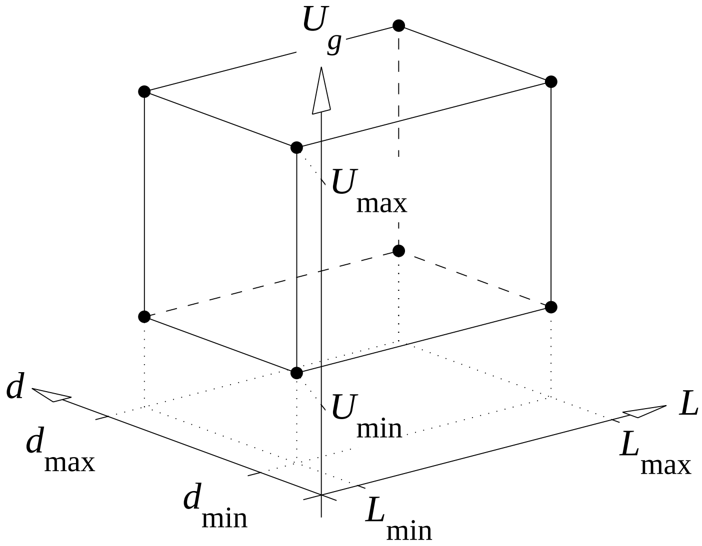

Figure 3. The region of interest, defined by upper and lower bounds on L, d and U_g. The corners, marked with ●, are used as training set.

When validating the model, the data points shown in figure 4 are chosen.

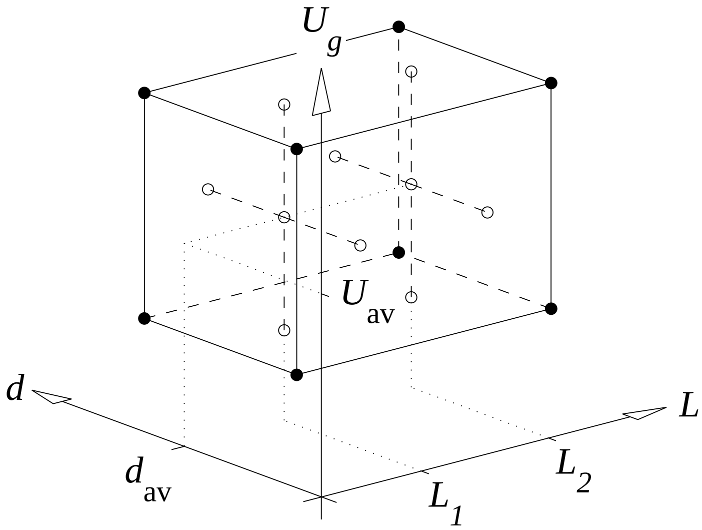

Figure 4. The points marked with ○ are used as test set, when validating the model.

The training and test sets are shown in tables 1 and 2, respectively.

Sample point	L	d	U_g	Ψ
t_1	$L_{\min}$	$d_{\min}$	$U_{\min}$	Ψ_1
t_2	$L_{\min}$	$d_{\min}$	$U_{\max}$	Ψ_2
t_3	$L_{\min}$	$d_{\max}$	$U_{\min}$	Ψ_3
t_4	$L_{\min}$	$d_{\max}$	$U_{\max}$	Ψ_4
t_5	$L_{\max}$	$d_{\min}$	$U_{\min}$	Ψ_5
t_6	$L_{\max}$	$d_{\min}$	$U_{\max}$	Ψ_6
t_7	$L_{\max}$	$d_{\max}$	$U_{\min}$	Ψ_7
t_8	$L_{\max}$	$d_{\max}$	$U_{\max}$	Ψ_8

Table 1. The training data set.

Sample point	L	d	U_g	Ψ
t_9	L_1	d_{av}	U_{av}	Ψ_9
t_{10}	L_1	d_{av}	$U_{\min}$	Ψ_{10}
t_{11}	L_1	d_{av}	$U_{\max}$	Ψ_{11}
t_{12}	L_1	$d_{\min}$	U_{av}	Ψ_{12}
t_{13}	L_1	$d_{\max}$	U_{av}	Ψ_{13}
t_{14}	L_2	d_{av}	U_{av}	Ψ_{14}
t_{15}	L_2	d_{av}	$U_{\min}$	Ψ_{15}
t_{16}	L_2	d_{av}	$U_{\max}$	Ψ_{16}
t_{17}	L_2	$d_{\min}$	U_{av}	Ψ_{17}
t_{18}	L_2	$d_{\max}$	U_{av}	Ψ_{18}

Table 2. The test data set.

4 Results

This section concerns results obtained by applying the proposed method to a test window. At current time, the data available for this profile is not completely consistent with the method described in this report. Other sample points than the ones described in section 3.2 have been used, and the data is not divided into training and test sets. Instead, all available data is used as training data. The optimal model parameters x^* have been estimated as an unconstrained solution to (3.3), which gives both positive and negative residuals. A future version of this report will show results consistent with the proposed method.

The optimal model parameters are:

$$x^* = \begin{bmatrix} 0.142233 \text{ m} \\ 0.324397 \text{ m} \\ 0.016726 \text{ m} \\ 0.767279 \text{ m}^2\text{K/W} \\ 0.149852 \text{ W/mK} \\ 0.753127 \text{ W/mK} \\ 1.113663 \text{ W/mK} \\ 0.116209 \text{ W/mK} \end{bmatrix} \tag{4.1}$$

The results are rounded to 6 digits. In table 3 are shown the available Therm calculations of Ψ, together with the residuals between the model and the data.

Sample point	$L\left[\frac{\text{W}}{\text{mK}}\right]$	d [mm]	$U_g\left[\frac{\text{W}}{\text{m}^2\text{K}}\right]$	$\Psi\left[\frac{\text{W}}{\text{mK}}\right]$	$r_i(x^*)\left[\frac{\text{W}}{\text{mK}}\right]$
t_1	0.133333	4	1.1	$3.62685\cdot10^{-2}$	$8.86917\cdot10^{-4}$
t_2	0.233333	4	1.1	$4.87297\cdot10^{-2}$	$4.23724\cdot10^{-4}$
t_3	0.4	4	1.1	$6.08245\cdot10^{-2}$	$1.20556\cdot10^{-4}$
t_4	0.733333	4	1.1	$7.27783\cdot10^{-2}$	$-2.42846\cdot10^{-4}$
t_5	1.733333	4	1.1	$8.48167\cdot10^{-2}$	$-8.86917\cdot10^{-4}$
t_6	0.171429	6	1.1	$4.64423\cdot10^{-2}$	$2.28961\cdot10^{-4}$
t_7	0.3	6	1.1	$5.98057\cdot10^{-2}$	$3.34297\cdot10^{-4}$
t_8	0.514286	6	1.1	$7.21824\cdot10^{-2}$	$6.10125\cdot10^{-4}$
t_9	0.942857	6	1.1	$8.38543\cdot10^{-2}$	$8.46534\cdot10^{-4}$
t_{10}	2.228571	6	1.1	$9.51032\cdot10^{-2}$	$8.25387\cdot10^{-4}$
t_{11}	0.133333	4	1.5	$3.23374\cdot10^{-2}$	$-8.86916\cdot10^{-4}$
t_{12}	0.233333	4	1.5	$4.37837\cdot10^{-2}$	$-6.56496\cdot10^{-4}$
t_{13}	0.4	4	1.5	$5.50044\cdot10^{-2}$	$-3.40490\cdot10^{-4}$
t_{14}	0.733333	4	1.5	$6.61406\cdot10^{-2}$	$-7.73590\cdot10^{-5}$
t_{15}	1.733333	4	1.5	$7.74460\cdot10^{-2}$	$-1.18398\cdot10^{-4}$
t_{16}	0.133333	4	2.0	$2.78233\cdot10^{-2}$	$8.86924\cdot10^{-4}$
t_{17}	0.233333	4	2.0	$3.80291\cdot10^{-2}$	$5.93164\cdot10^{-4}$
t_{18}	0.4	4	2.0	$4.81222\cdot10^{-2}$	$5.99702\cdot10^{-4}$
t_{19}	0.733333	4	2.0	$5.82716\cdot10^{-2}$	$7.42752\cdot10^{-4}$
t_{20}	1.733333	4	2.0	$6.86184\cdot10^{-2}$	$8.86929\cdot10^{-4}$

Table 3. The residuals between the model and the available data, for various sample points. The sample point with the largest absolute value of the residual is highlighted. All data is rounded to at most 6 digits.

The largest residual is obtained in the sample point t_{20}, with the absolute value $8.86929 \cdot 10^{-4}$ W/mK, which is clearly below the desired value.

Figure 5 shows a plot of the Therm calculations of Ψ, together with results obtained with the model (2.43), using the optimal parameters (4.1).

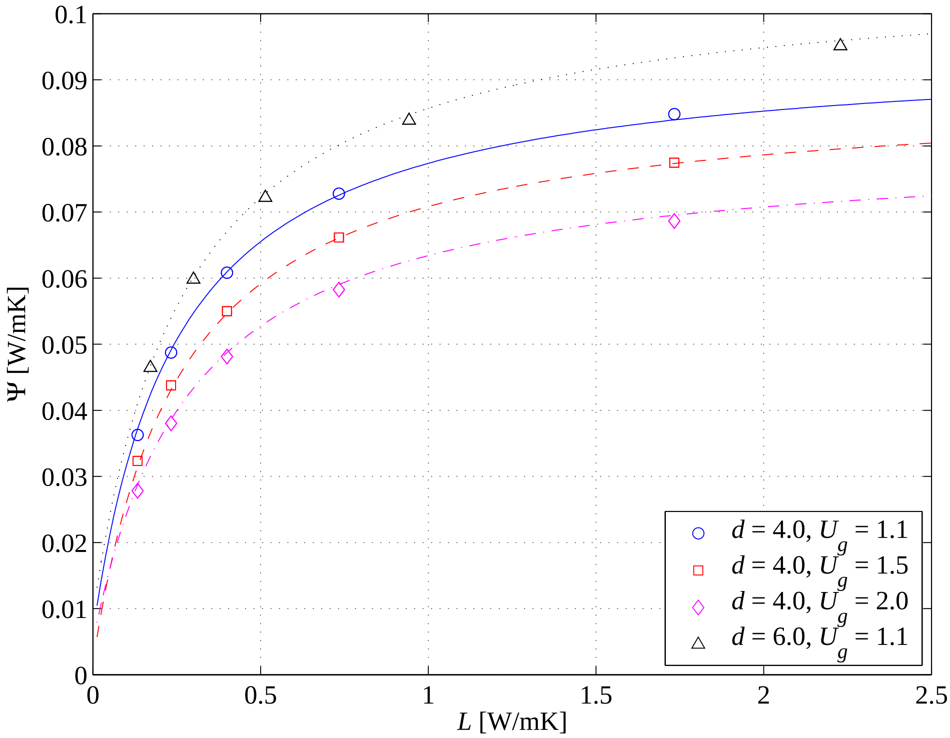

Figure 5. The points marked with ○, □, ◇ and △ are Therm calculations of Ψ *for the* L, d *and* U_g *values shown in table 3. The lines are obtained with the model* $\hat{\Psi}\left(x^*, t\right)$, *using the optimal model parameters* (4.1).

5 Implementation aspects

Using the proposed method requires implementations of the following functions:

1. A method for solving (3.3).
2. An implementation of the expressions (2.39), (2.40) and (2.43).

These functions can be implemented once and for all as either static or dynamic function libraries, which can be linked against the CAM[1] software, used by windows manufacturers. Once this is done, the end user does not have to worry about the details described in this report. This approach, however, involves the developers of the CAM software.

[1] Computer Aided Manufacturing.

Another possibility is to develop a separate application, which implements the above-mentioned functions. This approach does not involve the CAM software developers, but will probably not be as user friendly for the end user.

Assuming these details can be sorted out, window manufacturers has to carry out the following two steps in order to use the proposed method:

1. Estimate the optimal model parameters x^* by solving (3.3).
2. Given a set $t = \left[L, d, U_g \right]^{\mathrm{T}}$, the linear thermal transmittance Ψ can be estimated using the expressions (2.39), (2.40) and (2.43).

The first step requires detailed 2-dimensional heat flow calculations to be carried out in the training data set defined in Figure 3 and Table 1. The model parameters can be validated by comparing $\hat{\Psi}$-values obtained with the model, to detailed calculations of Ψ carried out in the test data set defined in Figure 4 and Table 2.

Once reliable model parameters have been estimated, they can be used for estimating Ψ for edge constructions, when combining a given frame profile with different glazing units, without carrying out detailed 2-dimensional heat flow calculations.

6 Future work

A future version of this report will include the following:

1. A section with nomenclature.
2. A clearer definition of the concepts L_1 and L_2-norm.
3. Investigating the possibilities for proving uniqueness of the solution to (3.3).
4. Reference regarding the exact penalty approach (3.4).
5. Results obtained by applying the method to a number of typical frame profiles.

7 References

[1] Kaj Madsen, Hans Bruun Nielsen and Ole Tingleff (1999), *Methods for Non-linear Least Squares Problems*, Lecture note H38 at Informatics and Mathematical Modelling, Technical University of Denmark.

[2] Matlab 6.5 and Matlab Symbolic Math Toolbox 2.1.3 (2002), The MathWorks, Inc., 3 Apple Hill Drive, Natick, MA 01760-2098, United States.

[3] Kaj Madsen (1975), *An Algorithm for Minimax Solution of Overdetermined Systems of Non-linear Equations*, Journal of the Institute of Mathematics and its Applications, vol. 16, pp. 321-328.